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Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

May / June - 2018

Mathematics : Course No. 3003

(Number Theory - I)

(New Course)

Faculty Code : 003

Subject Code : 1163003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Select the most appropriate answer for each of following :

(A) Using Euclidean algorithm we can find __________ of two integers.
(i) lcm (ii) gcd
(iii) product (iv) sum

(B) The number of primitive roots of 29 is __________
(i) 12 (ii) 13
(iii) 29 (iv) 28

(C) If $x^2+1\equiv 0 \pmod{p}$ has a solution then $p =$
(i) 59 (ii) 41
(iii) 79 (iv) 19

(D) If $x^2+1\equiv 0 \pmod{p}$ has a no solution then $p =$
(i) 13 (ii) 17
(iii) 29 (iv) 79

(E) If 59 divides a^2+b^2 then 79 divides __________
(i) a only (ii) b only
(iii) a and b both (iv) neither a nor b

(F) __________ is the consequence of Euler's theorem.

(i) Wilson's theorem (ii) Hensel lemma

(iii) Fermat's theorem (iv) Euclidean Algorithm

(G) The number of positive integers relatively prime to 1001 is __________

(i) 520 (ii) 1000

(iii) 720 (iv) 101

2 Attempt any **two** :

(A) Define the greatest common divisor of two integers. If g.c.d. of two integers a and b is g then prove that $\left(\frac{a}{g}, \frac{b}{g}\right) = 1$. **7**

(B) Suppose a and b are positive integers then prove that $(a,b)[a,b] = ab$. **7**

(C) State and prove Euclidean algorithm. **7**

3 All compulsory :

(A) Prove that there are infinitely many prime numbers. **7**

(B) Suppose p and q are distinct primes each of which divides n. Prove that pq divides n. **3**

(C) Use the Euclidean Algorithm to find the greatest common divisor of 1947 and 2017. **4**

OR

3 All compulsory :

(A) State and prove Euler's theorem. **7**

(B) Find all solutions of $x^2 \equiv -1 \pmod{17}$ in the complete residue system $\{0,1,2,....,16\}$. **4**

(C) Determine whether the congruence equation $x^2 \equiv -1 \pmod{59}$ has solution or not. **3**

4 Attempt any **two** :

(A) State and prove chinese remainder theorem. **7**

(B) Find the solutions of the following congruence equations if there is any. **7**

(i) $x^2 - 1 \equiv 0 \pmod{15}$

(ii) $x^2 + 1 \equiv 0 \pmod{125}$

(C) Suppose m is a positive integer such that $m = m_1 m_2$, $(m_1, m_2) = 1$, $(\phi(m_1), \phi(m_2)) \geq 2$. Prove that m has no primitive root. Give three positive integers of the above type which have no primitive roots. **7**

5 Do as directed. All are compulsory and each question carries two marks :

(a) Give the statement of Fermat's theorem

(b) Write the statement of Hensel's Lemma.

(c) Find the primitive roots of 3^2 and 5^2.

(d) Find the number of positive integers relatively prime to $1001 \times 25 \times 31$.

(e) Write the statement of Mobious inversion formula.

(f) Give all the positive divisors of p.q.r.s where p,q,r,s are distinct prime numbers.

(g) If $n \geq 1$ then what is the value of $\phi(7^n)$?
